

Supersymmetric M5-branes with H -field

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Abstract

In this paper we investigate the form of calibrated M5-branes in the presence of a nonvanishing 3-form field H . We discuss the influence of the H -field on the deformation of supersymmetric n -cycles (in particular SLAG submanifolds in \mathbb{R}^n). In addition we argue for a construction which relates calibrated M5-branes of different “curved” dimensions to each other.

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1 Introduction

Supersymmetric BPS brane solutions play a very important role in the description of string theory and gauge theories. Especially it became clear, that one can study gauge theories via various supersymmetric brane configurations embedded in flat spacetime. This approach is particularly attractive since a lot of informations about the nonperturbative gauge dynamics can be obtained by lifting the brane configurations to 11d M-theory [1], where branes preserving a certain amount of the 32 supercharges of M-theory are described by calibrated surfaces [2, 3, 4]. For example, gauge theories with eight unbroken supercharges ($N = 2$ supersymmetry in four dimensions) correspond to $SU(2)$ Special Lagrangian (SLAG) calibrations [1]. Similarly, gauge theories with four unbroken supercharges can be associated to $SU(3)$ -SLAG calibrations, the case which was discussed in [5]. However, unlike the supersymmetric 2-cycles, the supersymmetric 3-cycles and all higher odd dimensional SLAG cycles are very difficult to construct explicitly. Concrete applications requires a simple handling of such M5-brane solutions. Unfortunately for the $SU(3)$ -SLAG only extremely special solutions could be written down so far. Therefore one is looking for constructions which connect geometries of different dimensions to each other. In this context a very exciting construction exists [6],

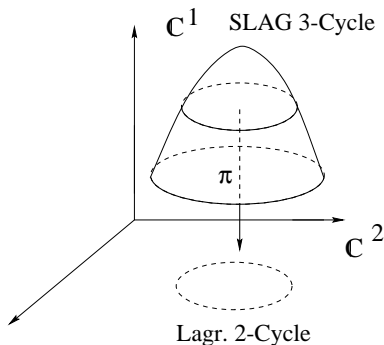


Fig.1. The projection principle

which connects Lagrangian submanifolds of different dimensions. The construction given there generates a smoothly varying phase α on the projected cycles which prevents it to be SLAG ($\alpha = \text{const}$) again (see Fig.1). We will show that supersymmetric solutions of the latter type exist where the non constant phase can be related to the 3 form field H which appears in the general equations of motions of the M5-brane^{*3}. In order to understand this “projection principle” we will analyze the complete BPS equations of the M5-brane, which includes additional terms with the capability to modify the differential

equations of the SLAG conditions. Such terms appear naturally after turning on the 3 form field H and after breaking further supersymmetries.

As a byproduct one can use these equations to discuss supersymmetric M5-brane solutions in a background including a constant 3 form field C of 11d SUGRA. This discussion is also motivated by the recent study of Yang-Mills theory in a noncommutative geometry [8]. In this way one might obtain nonperturbative informations about noncommutative Yang-Mills theories via H -deformed SW-curves.

For concreteness, we will focus on the cases of supersymmetric 2-cycles and show how the SLAG conditions are affected by the H -fields. The arguments are

^{*3}In a recent paper [7] similar modified geometries are investigated in IIA & IIB theory in the presence of F and B fields.

generic and can be generalized to all $SU(n)$ -SLAG cases as well.

The paper is organized as follows. In the next section we analyze the BPS equations of [4, 9] showing that SLAG submanifolds become LAG in the presence of the field H .

2 The modified Geometries

2.1 The BPS Equations

In static gauge the embedding of the M5-brane into flat 11d spacetime is realized by a map f , which describes the dependence of the transverse coordinates $X_{n'}$, $n' = 1, \dots, 5$ on the brane coordinates q_m , $m = 0, \dots, 5$. Furthermore there lives a two-form field B_{mn} on the six-dimensional worldvolume of the M5-brane with field strength $H = dB$. A nonlinear self duality constraint is realized on the field H , so that the anti self dual part can be computed from the self dual one:

$$H_{abc} = \frac{1}{Q} (h_{abc} + 2(kh)_{abc}).$$

k_a^b and Q are defined by $k_a^b = h_{acd}h^{bcd}$ and $Q = 1 - \frac{2}{3}trk^2$. The equations of motion of the M5-brane are obtained from the superembedding approach [10]. As it is shown there, in the transformation law of the residual spacetime supersymmetry a projector Γ appears,

$$\delta\Theta = (1 - \Gamma)\epsilon.$$

From this formula one can find supersymmetry preserving solutions by requiring $\delta\Theta \equiv 0$. Since the M5-brane breaks the $SO(1, 10)$ Lorentz invariance to a residual $SO(1, 5) \times SO(5)$ subgroup, the formulas are written in a well adapted form. In particular the 11d gamma matrices $\bar{\Gamma}_a$ are constructed out of $Spin(1, 5)$ and $Spin(5)$ gamma matrices by a general property of Clifford algebras. For the case at hand we obtain:

$$\begin{aligned} \mathcal{Cl}(\mathbb{R}^{1,5} \oplus \mathbb{R}^5) &\xrightarrow{\cong} \mathcal{Cl}(\mathbb{R}^{1,5}) \otimes \mathcal{Cl}(\mathbb{R}^5) \\ j(v \oplus w) &= v \otimes \mathbb{1} + \Gamma_7 \otimes w. \end{aligned} \quad (2.1)$$

Γ_7 is the chirality operator of the algebra $\mathcal{Cl}(\mathbb{R}^{1,5})$. Then the explicit coordinate expression of the last formula for a flat 11d SUGRA background as derived in [9] reads:

$$\begin{aligned} \hat{\delta}\Theta_{\beta}{}^j &= \frac{-1}{2 \det e} \epsilon^{\alpha i} \left\{ \partial_a X^{c'} (\gamma^a)_{\alpha\beta} (\gamma_{c'})_i{}^j - \frac{1}{3!} \partial_{a_1} X^{c'_1} \partial_{a_2} X^{c'_2} \partial_{a_3} X^{c'_3} (\gamma^{a_1 a_2 a_3})_{\alpha\beta} (\gamma_{c'_1 c'_2 c'_3})_i{}^j \right. \\ &\quad \left. + \frac{1}{5!} \partial_{a_1} X^{c'_1} \partial_{a_2} X^{c'_2} \partial_{a_3} X^{c'_3} \partial_{a_4} X^{c'_4} \partial_{a_5} X^{c'_5} (\gamma^{a_1 a_2 a_3 a_4 a_5})_{\alpha\beta} (\gamma_{c'_1 c'_2 c'_3 c'_4 c'_5})_i{}^j \right\} \\ &\quad - \frac{1}{2} \epsilon^{\alpha i} \left\{ -h^{m_1 m_2 m_3} \partial_{m_2} X^{c'_2} \partial_{m_3} X^{c'_3} (\gamma_{m_1})_{\alpha\beta} (\gamma_{c'_2 c'_3})_i{}^j - \frac{1}{3} h^{m_1 m_2 m_3} (\gamma_{m_1 m_2 m_3})_{\alpha\beta} \delta_i{}^j \right\}. \end{aligned} \quad (2.2)$$

Here γ^i are the chiral $Spin(1, 5)$ tangent space gamma matrices on the M5-brane and γ'^i the $Spin(5)$ gamma matrices of the transverse space. Without further specialisations the only solution is given by a flat M5-brane preserving one half of spacetime supersymmetry. On the other hand if there are some nontrivial identities of products of gamma matrices applied to the spinor ϵ one obtains differential equations which determine more general supersymmetric M5-brane solution. The amount of supersymmetry preserved by such a solution is determined by the number of relations imposed on the 11d gamma algebra.

2.2 The $SU(n)$ -SLAG calibrations

In the following we concentrate on those projectors which determine the Special Lagrangian geometries. They corresponds to the pattern of branes below (the bar denotes a negative sign of the eigenvalues of supersymmetries preserved by the brane),

	1	2		1'	2'
M5	1	2	3	4	5
$\overline{\text{M5}}$			3	4	5
				6	7

TABLE 1. $SU(2)$ -SLAG

	1	2	3		1'	2'	3'
M5	1	2	3	4	5		
$\overline{\text{M5}}$			3	4	5	6	7
M5		2		4	5	6	8

TABLE 2. $SU(3)$ -SLAG

which induce projectors of the type $\gamma^{ij} = -\gamma'^{ij}$. In the absence of the self dual 3 form h_{abc} these projectors imply the SLAG differential equations. The topology of the solution for $SU(n)$ -SLAG is $\mathbb{R}^{1,5-n} \times M_n$ with M_n the SLAG submanifold. The first three terms of eq. (2.2) were analyzed in [4]. For these types of geometries the authors found the BPS equations which for $SU(2)$ -SLAG and $SU(3)$ -SLAG read

$$\delta\Theta_{\beta^j} = \frac{-1}{2\det e} \epsilon^{\alpha i} \left\{ \frac{1}{2} \underbrace{\left[\partial_a X_c - \partial_c X_a \right]}_{\text{generic}} \gamma^a \gamma'^c + \left[\sum_a \partial_a X_a - \underbrace{\det(\partial X)}_{0 \text{ for } SU(2)} \right] \gamma^1 \gamma'^1 \right\} \quad (2.3)$$

The term in front of $\gamma^a \gamma'^c$ enforces the M5-brane to be a Lagrangian submanifold. That is a submanifold of half of the dimension of the embedding space on which the symplectic forms restricts to zero. The symplectic form is defined by $\omega(X_{\mathbb{R}}, Y_{\mathbb{R}}) = g(JX_{\mathbb{R}}, Y_{\mathbb{R}})$ with J the complex structure and g the Euclidean scalar product of the embedding space. Using the complex structure one can make a real vector $X_{\mathbb{R}}$ into a complex one $X_{\mathbb{C}}$. Since

$$\langle X_{\mathbb{C}}, Y_{\mathbb{C}} \rangle_{\mathbb{C}} = g(X_{\mathbb{R}}, Y_{\mathbb{R}}) + i \cdot \underbrace{\omega_{\mathbb{R}}(X_{\mathbb{R}}, Y_{\mathbb{R}})}_0$$

orthogonality in the real sense implies orthogonality in the complex sense and vice versa. Then for a manifold to be Lagrangian the complexified tangent

vectors must be complex orthonormalized. Locally the evaluation of the form

$$\Phi = (\tilde{e}_1 + i\tilde{e}_2) \wedge \dots \wedge (\tilde{e}_{2n-1} + i\tilde{e}_{2n})$$

on the tangent planes (\tilde{e}_i a dual basis of e_i , $Je_i = e_{i+1}$) measures the phase of the unitary matrix build out of these unitary tangent vectors. The second term in formula (2.3) can be understood as the condition $f^*\Im\Phi = 0$.

Whereas the geometry of the $SU(2)$ -SLAG calibration is nearly trivial, the $SU(3)$ generalization is highly complicated. To get a handle for the complicated case which are relevant for describing $N = 1$ chiral gauge theories [5] one is looking for constructive methods connecting BPS solutions of different dimensions to each other. As explained in [6] the simplest construction one can perform on a SLAG manifold only preserves the property of being a Lagrangian submanifold. Therefore we look for BPS solutions which are Lagrangian but do have a smoothly varying phase α , which is defined by $f^*\Phi = e^{i\alpha}\sqrt{g}dq^1 \wedge dq^2$ which leads to the formula

$$\tan \alpha = \frac{f^*\Im\Phi}{f^*\Re\Phi}. \quad (2.4)$$

The constraints following from supersymmetry are so restrictive that a non constant phase α can exist only if we turn on the field h_{abc} .

To indicate the origin of the interesting terms we consider the contribution of the 4th and 5th term of eq. (2.2) to the BPS equations. These terms contain the dependence on the field h_{abc} . Since h_{abc} is self dual the purely space like components can be computed from h_{0ij} . With $\epsilon^{0\dots 5} = -1$ this yields

$$h^{def} = \frac{1}{3!}\epsilon^{defrst}h_{rst} = \frac{1}{2}\underbrace{\epsilon^{def0st}}_{\text{Minkowskian}}h_{0st} = \frac{1}{2}\underbrace{\epsilon^{defst}}_{\text{Euclidean}}h_{0st}$$

which motivates the convenient abbreviations

$$\begin{aligned} F_{ab} &= h_{0ab} \in \Lambda^2(\mathbb{R}^5), \\ (*F)^{abc} &= \frac{1}{2}\epsilon^{abcde}h_{0de} \in \Lambda^3(\mathbb{R}^5). \end{aligned}$$

With these definitions the fourth term can be rewritten like:

$$\begin{aligned} \text{4th} &= -h^{m_1 m_2 m_3} \partial_{m_2} X^{c'_2} \partial_{m_3} X^{c'_3} (\gamma_{m_1})_{\alpha\beta} (\gamma_{c'_2 c'_3})_i{}^j \\ &= -h^{0ab} \partial_a X_{c_2} \partial_b X_{c_3} (\gamma_0)_{\alpha\beta} (\gamma'^{c_2 c_3})_i{}^j - h^{def} \partial_e X_{c_2} \partial_f X_{c_3} (\gamma_d)_{\alpha\beta} (\gamma'^{c_2 c_3})_i{}^j \\ &= F^{ab} \partial_a X_{c_2} \partial_b X_{c_3} \gamma_0 \gamma'^{c_2 c_3} - (*F)^{abc} \partial_a X_{c_2} \partial_b X_{c_3} \gamma_c \gamma'^{c_2 c_3}, \end{aligned}$$

and the last one simplifies to

$$\text{5th} = -\frac{1}{3}h^{m_1 m_2 m_3} (\gamma_{m_1 m_2 m_3})_{\alpha\beta} = 2F^{ab} \gamma_{0ab}.$$

Then the BPS equation gets modified in the following way:

$$\delta\Theta_\beta{}^j = \dots - \frac{1}{2}\epsilon^{\alpha i} \left\{ F^{ab} \partial_a X_{c_2} \partial_b X_{c_3} \gamma_0 \gamma'^{c_2 c_3} - (*F)^{abc} \partial_a X_{c_2} \partial_b X_{c_3} \gamma_c \gamma'^{c_2 c_3} + 2F^{ab} \gamma_{0ab} \right\}$$

If one does not introduce additional projectors, which further reduce the amount of supersymmetry, the last equation states that the cycle remains $SU(n)$ -SLAG but will be deformed by the field h_{abc} . Technically speaking the moduli of the cycle become functions of the field h_{abc} and are partially eliminated in favour of the degrees of freedom of the field h_{abc} . On the other hand breaking SUSY further by additional projectors the $SU(n)$ -SLAG conditions will be affected as we will discuss now.

2.3 Investigation of the basic example

Now we specialize all considerations for the $SU(2)$ -SLAG example as given in TABLE 1 which contains the generic behaviour. One can use a duality relation of the chiral 6d gamma matrices,

$$\gamma^{a_1 \dots a_n} = - \frac{1}{(6-n)!} (-1)^{\frac{n(n+1)}{2}} \epsilon^{a_1 \dots a_n a_{n+1} \dots a_6} \gamma_{a_{n+1} \dots a_6}, \quad (2.5)$$

to further simplify this expression. We want to put as many as possible γ -terms into the form $\gamma_0 \gamma'^{c_1 c_2}$. Writing out the corresponding expression for the 2-cycle one obtains

$$(*F)^{abc} \partial_a X_{c_2} \partial_b X_{c_3} \gamma_c \gamma'^{c_2 c_3} = 2(*F)^{12c} \partial_1 X_{[1} \partial_2 X_{2]} \gamma_c \gamma'^{12}$$

This has to be discussed for $c = 3, \dots, 5$. With the help of the projectors and the 6d duality relation eq. (2.5) we may compute,

$$\gamma_3 \gamma'^{12} \epsilon = - \gamma_0 \gamma'^{12} \epsilon,$$

and $F^{123} = F^{45}$. Similarly we may conclude $\gamma_{045} = - \gamma_0 \gamma'^{12}$. Then the BPS equations read

$$\begin{aligned} \delta \Theta_{\beta}^j = & -\frac{1}{2} \det(e^{-1}) \epsilon^{\alpha i} \left\{ \frac{1}{2} \left[\partial_a X_c - \partial_c X_a \right] \gamma^a \gamma'^c + \left[\sum_a \partial_a X_a \right] \gamma^1 \gamma'^1 \right\} \\ & - \frac{1}{2} \epsilon^{\alpha i} \left\{ 2(F^{12} + F^{45}) (\partial_1 X_1 \partial_2 X_2 - \partial_2 X_1 \partial_1 X_2) - 4(F^{12} + F^{45}) \right\} \gamma_0 \gamma'^{12} \\ & + \dots \end{aligned} \quad (2.6)$$

where the dots are the terms for $c = 4, 5$ implying that the corresponding components of F^{ab} vanish. As it stands the cycle remains $SU(2)$ -SLAG and preserves 1/4 of spacetime supersymmetry. In fact for this simple case the closure of H implies the absence of the field h_{abc} . A completely different behaviour arises after breaking further supersymmetries. Concretely we will break supersymmetry by imposing additional projectors which can be translated to the following modified brane picture below [9]:

	1	2				1'	2'
M5	0	1	2	3	4	5	
$\overline{\text{M5}}$	0			3	4	5	6
M2	0	1					7

TABLE 3. Brane configuration

From the additional “M2-brane projector” $\bar{\Gamma}_{017} \epsilon = \eta \epsilon$ ($\bar{\Gamma}_a$ 11d tangent frame gamma matrices, η a sign) one obtains

$$\bar{\Gamma}_{067} \epsilon = -\eta \bar{\Gamma}_{16} \epsilon$$

and by the construction of $\bar{\Gamma}_a$ (see eq. 2.1) this is identical to

$$\gamma_0 \gamma'^{12} \epsilon = \eta \gamma^1 \gamma'^1 \epsilon.$$

This equality generates an inflow from the purely h -field terms to the term proportional to $\gamma^1 \gamma'^1$ and modifies the SLAG condition while preserving the Lagrangian property^{*4}. If one writes out the resulting BPS equations, one finds:

$$\begin{aligned} \delta \Theta_{\beta}^j &= -\frac{1}{2} \det(e^{-1}) \epsilon^{\alpha i} \left\{ \frac{1}{2} \left[\partial_1 X_2 - \partial_2 X_1 \right] \gamma^1 \gamma'^2 + \left[\sum_a \partial_a X_a \right] \gamma^1 \gamma'^1 \right\} \\ &\quad - \frac{1}{2} \epsilon^{\alpha i} \left\{ 2(F^{12} + F^{45}) (\partial_1 X_1 \partial_2 X_2 - \partial_2 X_1 \partial_1 X_2) - 4(F^{12} + F^{45}) \right\} \underbrace{\gamma_0 \gamma'^{12}}_{\eta \gamma^1 \gamma'^1} \\ &= -\frac{1}{2} \det(e^{-1}) \epsilon^{\alpha i} \left\{ \frac{1}{2} \left[\partial_1 X_2 - \partial_2 X_1 \right] \gamma^1 \gamma'^2 + \left[\sum_a \partial_a X_a + \eta \cdot \det(e) \cdot \right. \right. \\ &\quad \left. \left. \left(2(F^{12} + F^{45}) (\partial_1 X_1 \partial_2 X_2 - \partial_2 X_1 \partial_1 X_2) - 4(F^{12} + F^{45}) \right) \right] \gamma^1 \gamma'^1 \right\} \end{aligned}$$

We can solve for the F_{ab} :

$$F^{12} + F^{45} = \frac{\eta}{2 \det(e)} \frac{\sum_a \partial_a X_a}{2 - (\partial_1 X_1 \partial_2 X_2 - \partial_2 X_1 \partial_1 X_2)} \quad (2.7)$$

In addition to this constraints one also has to ensure the closure of the spacetime 3 form $H_{ijk} = e_i^a e_j^b e_k^c H_{abc}$. The simplest way to satisfy the closure of H_{ijk} is to take the components F^{12} and F^{45} to be constant.

Before we discuss the consequence of this choice let us remind some of the geometrical formulas. The pullbacks of the embedding space differential forms

^{*4} Instead of including the M2-brane projector $\bar{\Gamma}_{017} \epsilon = \eta \epsilon$ one could choose $\bar{\Gamma}_{016} \epsilon = \kappa \epsilon$, which modifies the Lagrangian condition but preserves the other equation. But this case is not so well suited for our purposes but seems to be the generic one for higher dimensions.

ω , $\Re \Phi$ and $\Im \Phi$ under the map f read

$$\begin{aligned} f^* \omega &= (\partial_2 X_1 - \partial_1 X_2) dq^1 \wedge dq^1, \\ f^* \Re \Phi &= (1 - \partial_1 X_1 \partial_2 X_2 + \partial_2 X_1 \partial_1 X_2) dq^1 \wedge dq^2, \\ f^* \Im \Phi &= (\partial_1 X_1 + \partial_2 X_2) dq^1 \wedge dq^2. \end{aligned}$$

We will need the prefactors in front of the differentials which we denote by $[f^* \omega]$ and so on. The induced metric on the embedded submanifold M^2 is given by

$$g_{ij} = \begin{pmatrix} 1 + (\partial_1 X_1)^2 + (\partial_1 X_2)^2 & \partial_1 X_1 \partial_2 X_1 + \partial_1 X_2 \partial_2 X_2 \\ \partial_1 X_1 \partial_2 X_1 + \partial_1 X_2 \partial_2 X_2 & 1 + (\partial_2 X_1)^2 + (\partial_2 X_2)^2 \end{pmatrix}$$

and the following algebraic identity

$$\det g = [f^* \Im \Phi]^2 + [f^* \Re \Phi]^2 + [f^* \omega]^2$$

holds. Combining this identity with the BPS equation $f^* \omega = 0$ and the BPS equation (2.7) one computes for the phase (eq. 2.4) the expression below,

$$\tan \alpha = 2\eta (F^{12} + F^{45}) \sqrt{\frac{(1 + [f^* \Re \Phi])^2}{1 - 4(F^{12} + F^{45})^2 (1 + [f^* \Re \Phi])^2}},$$

which is generically not constant. Therefore the cycle is $SU(2)$ -SLAG only if the two moduli F^{12} and F^{45} vanish. Furthermore for non vanishing F^{12} and F^{45} the cycle is not anymore SLAG but still a Lagrangian submanifold^{*5}.

3 Conclusions

We have shown that for the construction of supersymmetric solutions the field h_{abc} can be used in two alternative directions. Turning on the field h_{abc} without further breaking of supersymmetries does not change the class of calibrated submanifolds one started with but only deforms the cycle inside this class. Alternatively one can break additional supersymmetries through further projectors, which would lead to supersymmetric solutions which are not covered by the list of standard calibrations. Here we studied the example on an $1/8$ supersymmetric 2-cycle which is still Lagrangian. Following the construction given in [6] we conjecture that the Lagrangian 2-cycle can be obtained by projecting a $SU(3)$ -SLAG cycle into a four dimensional subspace. Partial evidence for this conjecture was given in the paper.

The limitations of our calculations are mainly concerned with the restriction to a flat metric SUGRA background but this can be improved in principle. Partial results are available for nonflat backgrounds [11, 7]. Both approaches do have close relations but are difficult to adapt.

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^{*5}For the choice of projectors depicted in TABLE 3.

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